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A NEW APPROACH FOR NIZHNIK-NOVIKOV-VESELOV SYSTEM

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Abstract. Studies in recent years have examined the analytical solutions of nonlinear evolution equations (NLEEs). These equations are handled by using a lot of methods. Especially, hyperbolic function solutions, soliton solutions and others have been revealed for these NLEEs. A powerful technique is used to attain some exact solutions of Nizhnik-Novikov-Veselov (NNV) system. Hyperbolic function solutions and dark soliton solutions of this system are constructed via modified $\exp(-\Omega(\xi))$ -expansion function method (MEFM). Then, results that we obtained are submitted. Moreover, we draw 2D and 3D surfaces of some analytical solutions that we procured with the help of this method.

Keywords: Hyperbolic function solutions, soliton solutions, Nizhnik-Novikov-Veselov system, modified $\exp(-\Omega(\xi))$ - expansion function method.

AMS Subject Classification: 35-04, 35C08, 35N05, 68N15.

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1 Introduction

Nonlinear evolution equations (NLEEs) are comprehensively analyzed in establishing of physical circumstances in various areas of sciences. Therefore, exact solutions for NLEEs have a significant role in many circumstances such as viscoelasticity, economics, signals processing, hydrodynamics, plasma physics, and so on.

In recent years, many scientists have submitted various methods to ascertain analytical solutions of NLEEs (Foroutan et al., 2018; Ravi et al., 2017; Bibi & Mohyud-Din, 2014; Demiray & Bulut, 2016; Demiray & Bulut, 2017). In this work, the MEFM Baskonus et al. (2016) will be performed to obtain exact solutions NNV system.

We consider NNV system (Zhao et al., 2004; Dai et al., 2006; Tang et al., 2012; Tasbozan et al., 2013),

$$\begin{aligned} & u_t + au_{xxx} + bu_{yyy} - 3a(uv)_x - 3b(uw)_y = 0, \\ & u_x = v_y, u_y = w_x, \end{aligned} \tag{1}$$

where a and b are nonzero constants. Eq. (1) which is a (2+1) dimensional integrable system is acknowledged as isotropic Lax extension of the familiar (1+1) dimensional KdV equation Dai et al. (2006). Besides, Eq. (1) is assumed as a model for an incompressible fluid Tang et al. (2012).

In this work, our mission is to get exact solutions of NNV system. In Sec. 2, we clarify statement of MEFM. In Sec. 3, we get exact solutions of this system. In Sec. 4, we notify the results that we obtained by using this method.

2 General Structure of Method

We handle PDE as follows:

$$P(u, u_x, u_y, u_t, u_{xx}, u_{yy}, u_{tt}, \cdots) = 0, \tag{2}$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u(x, y, t)$ and its derivatives, in which the highest order derivatives and nonlinear terms are included and the subscripts demonstrate the partial derivatives. The fundamental stages of the method are defined as follows:

Step 1: We get traveling transformation

$$u(x, y, t) = U(\xi), \xi = x + y + ct. \tag{3}$$

By means of Eq. (3), we can reduce Eq. (2) to a nonlinear ordinary differential equation (NODE) given by:

$$NODE(U, U', U'', U''', \cdots) = 0, \tag{4}$$

where $NODE$ is a polynomial of U and its derivatives and the superscripts exhibit the ordinary derivatives with respect to ξ.

Step 2: Presume the traveling wave solution of Eq. (4) can be indicated as follows:

$$U(\xi) = \frac{\sum_{i=0}^{N} A_i \left[\exp\left(-\Omega(\xi)\right)\right]^i}{\sum_{j=0}^{M} B_j \left[\exp\left(-\Omega(\xi)\right)\right]^j} = \frac{A_0 + A_1 \exp(-\Omega) + \cdots + A_N \exp\left(N(-\Omega)\right)}{B_0 + B_1 \exp(-\Omega) + \cdots + B_M \exp\left(M(-\Omega)\right)}, \tag{5}$$

where $A_i, B_j, (0 \leq i \leq N, 0 \leq j \leq M$ are constants to be described later, such that $A_N \neq 0, B_M \neq 0$, and $\Omega = \Omega(\xi)$ is solution of the following ordinary differential equation:

$$\Omega'(\xi) = \exp\left(-\Omega(\xi)\right) + \mu \exp\left(\Omega(\xi)\right) + \lambda. \tag{6}$$

The solution families of Eq. (6) can be shown as follows:

Family 1: If $\mu \neq 0, \lambda^2 - 4\mu > 0$,

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{7}$$

Family 2: If $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$\Omega(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{8}$$

Family 3: If $\mu = 0, \lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$\Omega(\xi) = -\ln\left(\frac{\lambda}{\exp\left(\lambda(\xi + E)\right) - 1}\right). \tag{9}$$

Family 4: If $\mu \neq 0, \lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$\Omega(\xi) = \ln\left(-\frac{2\lambda(\xi + E) + 4}{\lambda^2(\xi + E)}\right). \tag{10}$$

Family 5: If $\mu = 0, \lambda = 0$, and $\lambda^2 - 4\mu = 0$,

$$\Omega(\xi) = \ln(\xi + E). \tag{11}$$

The positive integers N and M can be determined by paying regard to the homogeneous balance principle in Eq. (5).

Step 3: Replacing Eqs. (6) and (7-11) into Eq. (5), we ascertain a polynomial of $\exp(-\Omega(\xi))$. We stabilize all the coefficients of same power of $\exp(-\Omega(\xi))$ to zero. This operation determines a system of equations which can be unfastened to reach $A_0, A_1, A_2, \cdots, A_N, B_0, B_1, B_2, \cdots,$ B_M, E, λ, μ by the way of Wolfram Mathematica 9. Inserting the values of these constants into Eq. (5), the general solutions of Eq. (5) supply the determination of the solution of Eq. (1).

3 MEFM for Nizhnik-Novikov-Veselov System

Firstly, we get the travelling wave transformation,

$$u(x,y,t) = u(\xi), v(x,y,t) = v(\xi), w(x,y,t) = w(\xi), \xi = kx + my - nt \tag{12}$$

where k, m and n are nonzero constants.

Putting Eqs. (13-15) into Eq. (1),

$$u_t = -nu', u_x = ku', u_y = mu' \tag{13}$$

$$v_x = kv', v_y = mv', w_x = kw', v_y = mw' \tag{14}$$

$$u_{xxx} = k^3 u''', u_{yyy} = m^3 u''', \tag{15}$$

we obtain following system

$$-nu' + (ak^3 + bm^3)u''' - 3ak(uv)' - 3bm(uw)' = 0, \tag{16}$$

$$ku' = mv', mu' = kw'. \tag{17}$$

Taking the integral of Eq. (17) with regard to ξ and ignoring the integration constant, we procure

$$v(\xi) = \frac{k}{m}u(\xi), w(\xi) = \frac{m}{k}u(\xi). \tag{18}$$

Inserting Eq. (18) into Eq. (16), we have

$$-nmku + mk(ak^3 + bm^3)u'' - (3ak^3 + 3bm^3)u^2 = 0. \tag{19}$$

Benefiting from the balance principle, we procure

$$N = M + 2. \tag{20}$$

When we get $M = 1$ so $N = 3$, we can write following equations:

$$U = \frac{A_0 + A_1 \exp(-\Omega) + A_2 \exp(2(-\Omega)) + A_3 \exp(3(-\Omega))}{B_0 + B_1 \exp(-\Omega)}, \tag{21}$$

$$U' = \frac{[A_1 \exp(-\Omega)(-\Omega') + A_2 \exp(2(-\Omega))(-2\Omega') + A_3 \exp(3(-\Omega))(-3\Omega')]}{[B_0 + B_1 \exp(-\Omega)]}$$

$$-\frac{[A_0 + A_1 \exp(-\Omega) + A_2 \exp(2(-\Omega)) + A_3 \exp(3(-\Omega))][B_1 \exp(-\Omega)(-\Omega')]}{[B_0 + B_1 \exp(-\Omega)]^2} = \frac{\Upsilon}{\Psi}, \tag{22}$$

$$U'' = \frac{\Upsilon'\Psi - \Upsilon\Psi'}{\Psi^2},$$

$$\vdots$$

where $A_3 \neq 0$ and $B_1 \neq 0$. When we pay attention to Eq.(21) and Eq.(22) in Eq.(19), it supplies us the following coefficients:

Case 1:

$$\begin{aligned} & A_0 = \tfrac{\mu A_3 B_0}{B_1}, A_1 = A_3\left(\mu + \tfrac{\lambda B_0}{B_1}\right), A_2 = A_3\left(\lambda + \tfrac{B_0}{B_1}\right), A_3 = A_3, \\ & k = \tfrac{A_3}{2mB_1}, n = \tfrac{(\lambda^2-4\mu)(aA_3^3+8bm^6B_1^3)}{8m^3B_1^3}, B_0 = B_0, B_1 = B_1, \lambda = \lambda, \mu = \mu. \end{aligned} \tag{23}$$

Case 2:

$$\begin{aligned} & A_0 = 2km\mu B_0, A_1 = \tfrac{B_0(A_2-2kmB_0)}{B_1} + 2km\mu B_1, A_2 = A_2, A_3 = 2kmB_1, \\ & \lambda = \tfrac{A_2-2kmB_0}{2kmB_1}, n = \tfrac{(ak^3+bm^3)\left((A_2-2kmB_0)^2-16k^2m^2\mu B_1^2\right)}{4k^2m^2B_1^2}, B_0 = B_0, B_1 = B_1, \mu = \mu. \end{aligned} \tag{24}$$

Case 3:

$$\begin{aligned} & A_0 = \tfrac{(\lambda^2+2\mu)A_3B_0}{6B_1}, A_1 = \tfrac{A_3}{6}\left(\lambda^2 + 2\mu + \tfrac{6\lambda B_0}{B_1}\right), A_2 = A_3\left(\lambda + \tfrac{B_0}{B_1}\right), A_3 = A_3, \\ & k = \tfrac{A_3}{2mB_1}, n = -\tfrac{(\lambda^2-4\mu)(aA_3^3+8bm^6B_1^3)}{8m^3B_1^3}, B_0 = B_0, B_1 = B_1, \lambda = \lambda, \mu = \mu. \end{aligned} \tag{25}$$

Embedding Eq. (23) together with Eqs. (3) and (7) in Eq. (21), we ascertain new hyperbolic function solutions of Eq. (1),

$$u_1(x,y,t) = \frac{\mu(-\lambda^2+4\mu)A_3}{\left(\lambda\cosh[f(x,y,t)] + \sqrt{\lambda^2-4\mu}\sinh[f(x,y,t)]\right)^2 B_1}, \tag{26}$$

$$v_1(x,y,t) = \frac{\mu(-\lambda^2+4\mu)kA_3}{\left(\lambda\cosh[f(x,y,t)] + \sqrt{\lambda^2-4\mu}\sinh[f(x,y,t)]\right)^2 mB_1}, \tag{27}$$

$$w_1(x,y,t) = \frac{\mu(-\lambda^2+4\mu)mA_3}{\left(\lambda\cosh[f(x,y,t)] + \sqrt{\lambda^2-4\mu}\sinh[f(x,y,t)]\right)^2 kB_1}, \tag{28}$$

where

$$f(x,y,t) = \frac{1}{2}\sqrt{\lambda^2-4\mu}\left[E + my + \frac{xA_3}{2mB_1} - \frac{t(\lambda^2-4\mu)(aA_3^3+8bm^6B_1^3)}{8m^3B_1^3}\right],$$

and $\mu \neq 0, \lambda^2 - 4\mu > 0$.

Substituting Eq. (24) together with Eqs. (3) and (7) in Eq. (21), we reach new hyperbolic function solutions of Eq. (1),

$$u_2(x,y,t) = \frac{2km\mu\left[-(A_2-2kmB_0)^2 + 16k^2m^2\mu B_1^2\right]\sec h^2[g(x,y,t)]}{\left[A_2 - 2kmB_0 + 2kmB_1\sqrt{-4\mu + \frac{(A_2-2kmB_0)^2}{4k^2m^2B_1^2}}\tanh[g(x,y,t)]\right]^2}, \tag{29}$$

$$v_2(x,y,t) = \frac{2k^2\mu\left[-(A_2-2kmB_0)^2 + 16k^2m^2\mu B_1^2\right]\sec h^2[g(x,y,t)]}{\left[A_2 - 2kmB_0 + 2kmB_1\sqrt{-4\mu + \frac{(A_2-2kmB_0)^2}{4k^2m^2B_1^2}}\tanh[g(x,y,t)]\right]^2}, \tag{30}$$

$$w_2(x,y,t) = \frac{2m^2\mu\left[-(A_2-2kmB_0)^2 + 16k^2m^2\mu B_1^2\right]\sec h^2[g(x,y,t)]}{\left[A_2 - 2kmB_0 + 2kmB_1\sqrt{-4\mu + \frac{(A_2-2kmB_0)^2}{4k^2m^2B_1^2}}\tanh[g(x,y,t)]\right]^2}, \tag{31}$$

where

$$g(x,y,t) = \frac{1}{2}\sqrt{-4\mu + \frac{(A_2-2kmB_0)^2}{4k^2m^2B_1^2}} \times$$

$$\times\left[E + kx + my + 4(ak^3 + bm^3)t\mu - \frac{(ak^3 + bm^3)t(A_2 - 2kmB_0)^2}{4k^2m^2B_1^2}\right]$$

and

$$-4\mu + \frac{(A_2 - 2kmB_0)^2}{4k^2m^2B_1^2} > 0.$$

Embedding Eq. (25) together with Eqs. (3) and (7) in Eq. (21), we detect new dark soliton solutions of Eq. (1),

$$u_3(x,y,t) = \frac{(\lambda^2 - 4\mu)A_3\left[\lambda^2 - 6\mu + 2\lambda\sqrt{\lambda^2 - 4\mu}\tanh\left[h(x,y,t)\right] + (\lambda^2 + 2\mu)\tanh^2\left[h(x,y,t)\right]\right]}{6B_1\left[\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left[h(x,y,t)\right]\right]^2}, \quad (32)$$

$$v_3(x,y,t) = \frac{k(\lambda^2 - 4\mu)A_3\left[\lambda^2 - 6\mu + 2\lambda\sqrt{\lambda^2 - 4\mu}\tanh\left[h(x,y,t)\right] + (\lambda^2 + 2\mu)\tanh^2\left[h(x,y,t)\right]\right]}{6mB_1\left[\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left[h(x,y,t)\right]\right]^2}, \quad (33)$$

$$w_3(x,y,t) = \frac{m(\lambda^2 - 4\mu)A_3\left[\lambda^2 - 6\mu + 2\lambda\sqrt{\lambda^2 - 4\mu}\tanh\left[h(x,y,t)\right] + (\lambda^2 + 2\mu)\tanh^2\left[h(x,y,t)\right]\right]}{6kB_1\left[\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left[h(x,y,t)\right]\right]^2}, \quad (34)$$

where

$$h(x,y,t) = \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left[E + my + \frac{xA_3}{2mB_1} + \frac{t(\lambda^2 - 4\mu)(aA_3^3 + 8bm^6B_1^3)}{8m^3B_1^3}\right]$$

and $\mu \neq 0, \lambda^2 - 4\mu > 0$.

Remark 1. *The obtained results are validated to control the accuracy. Namely, the computer simulation results are affirmative. The whole solutions of Eq. (1) are novel and are not acquired in former reports.*

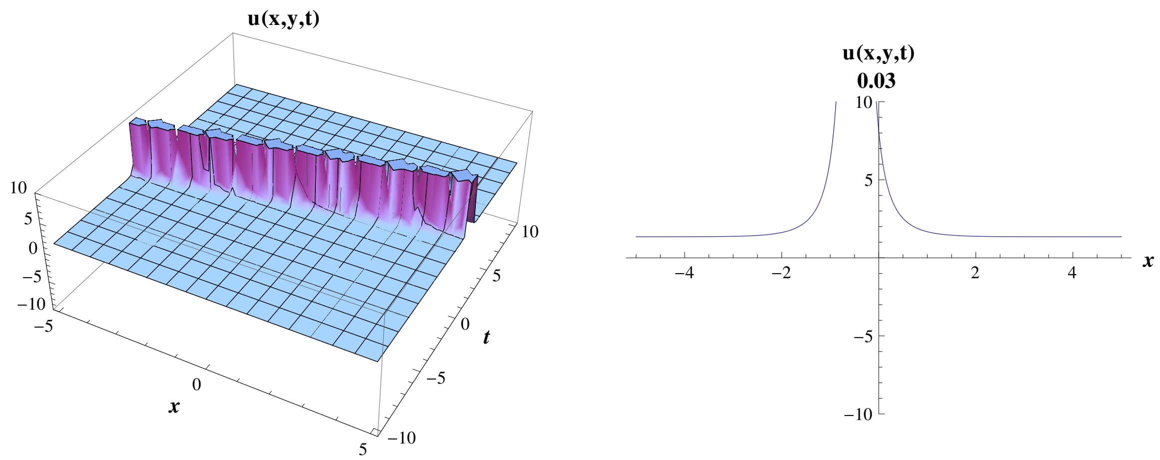

Figure 1: Graph of Eq. (32) for $\lambda = 0.1, \mu = -0.2, k = 1, m = 2, a = -1, b = 1, A_3 = 2,$ $B_1 = 0.2, E = 0.5, y = 0.3, -5 < x < 5, -10 < t < 10$ and $t = 0.03$ for 2D surface.

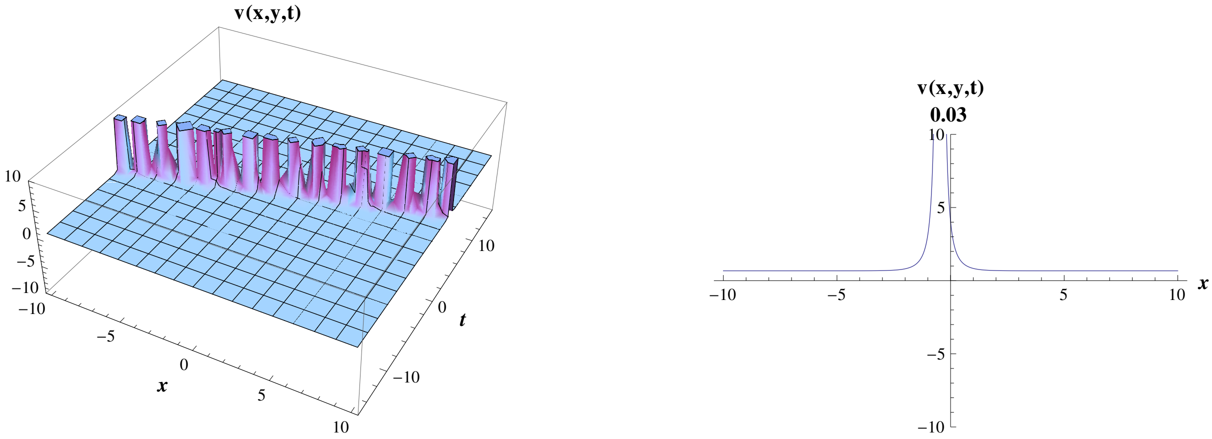

Figure 2: Graph of Eq. (33) for $\lambda = 0.1, \mu = -0.2, k = 1, m = 2, a = -1, b = 1, A_3 = 2,$ $B_1 = 0.2, E = 0.5, y = 0.3, -10 < x < 10, -15 < t < 15$ and $t = 0.03$ for 2D surface.

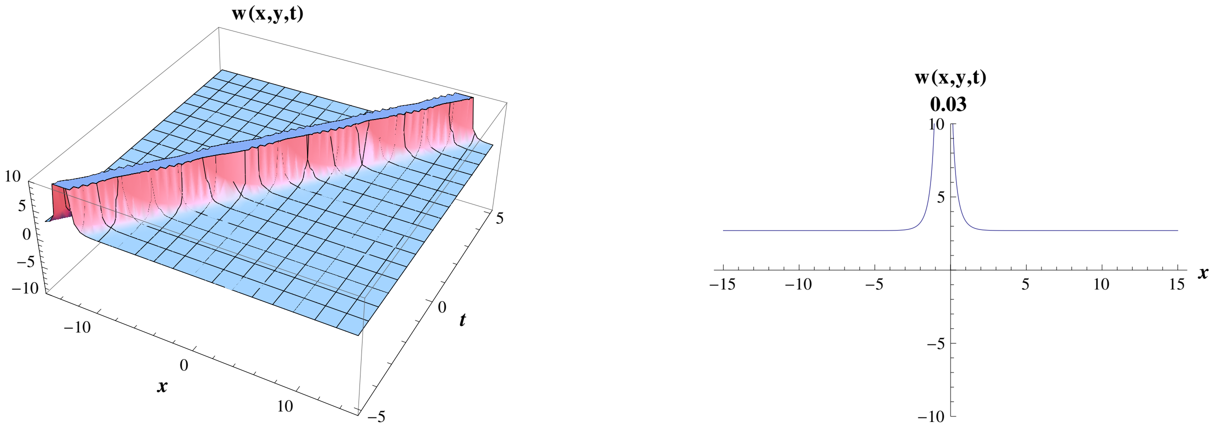

Figure 3: Graph of Eq. (34) for $\lambda = 0.1, \mu = -0.2, k = 1, m = 2, a = -1, b = 1, A_3 = 2,$ $B_1 = 0.2, E = 0.5, y = 0.3, -15 < x < 15, -5 < t < 5$ and $t = 0.03$ for 2D surface.

4 Conclusion

In this work, we tackle exact solutions of NNV system by using MEFM. Then, we draw graphics of some exact solutions of NNV system by the way of Mathematica. The pictographic representations overtly prove effectiveness of proposed method.

The attracted results supply us a strong method to create exact solutions by means of MEFM. To demonstrate the powerful mathematical device of this method, the solutions of NNV system are considered and new solutions of this system are reached. Our inferences exhibit that this method is considerably influent in terms of procuring new solutions of NLEEs.

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